M2 MMMEF – UE 1
Content of the courses

## Stochastic Calculus 1

Stochastic calculus and in particular Itô's integral is the main conceptual tool to deal with the stochastic processes appearing in finance. This course aims to introduce this integral and its mathematical foundations.

After a reminder on probability theory and on conditional expectation, this course will cover the following concepts:

- Brownian motion and its properties.
- Continuous time martingales, Stopping theorems, Doob's inequality.
- Uniformly integrable martingales.
- Itô's integral with respect to a Brownian motion.
- Local martingales, semi-martingales, and their quadratic variation.
- Itô's integral with respect to semi-martingales.
- Itô's formula.

## Arbitrage Theory

This course is an introduction to the notion of arbitrage in Finance in the discrete time setting. We show in particular the fundamental theorem of asset pricing linking arbitrage opportunities and equivalent martingale measure. In the framework of complete markets we study how to deduce the price of European derivatives from the underlying dynamics and how to hedge associated risks. We generalize the preceding framework considering the problems of:

- Evaluation of American options and hedging using strategies with consumption.
- Over-replication in incomplete markets.
- Dividends and transaction costs.

## Market risk measures

This course presents the performance and risk measures that are used in the asset management industry and in the risk department of investment banks. In particular, we discuss in depth Sharpe Ratio, Value at Risk and Expected Shortfall, along with their mathematical properties. The course requires advance knowledge in probability and statistics but many concepts are recalled such as quantiles, Fréchet bounds for correlations, copulas, etc.

## Introduction to finance

This course is an introduction to finance with a focus on fixed income products. It goes along the following line:

- Some words on the interest rates building up
- Future and Present Value
- Some financial products (Bonds, swaps and others)

- Duration and convexity of a bond
- Zero coupon curve
- Caps/Floors; swaptions

Financial time series analysis

This course is an introduction to financial time series analysis with a focus on Hawkes processes. It goes along the following line:

- Stochastic processes: first definitions and properties.
    - The classical Poisson process
    - Thinning and superposition
    - Law of Large Numbers and Central Limit Theorem.
    - Poisson processes with inhomogeneous rates.
- One-dimensional linear Hawkes process: definition, construction, properties, non-explosion, stationarity, law of large numbers, mean number of jumps, empirical covariation across time scales, longtime behavior.
- Multivariate linear Hawkes processes; Clustering representation.
- Application to financial time series.

Python and advanced Python for optimization and Finance

This Python course starts with the classical structures of the Python language. Many libraries are presented, including NumPy, SciPy, Pandas and TensorFlow.

The students must have some experience in programming.

Combinatorial optimization

The lecture includes three parts. Depending on the students' backgrounds, emphasis will be placed on one or other of these parts:

- Graph Theory and algorithmic applications: Shortest path problem, Maximum flow, minimum cut, Ford-Fulkerson algorithm, Graph coloring, cliques and independent sets, Matchings in bi-partite graphs, hungarian method, General matchings, Edmonds algorithm.
- Computational complexity: P, NP classes, Cook's Theorem, NP-complete problems, Karp and Turing reductions, Examples of NP-complete problems.
- Integer linear programming: Simplex method, Sufficient conditions for linear programs integrality, totally unimodular matrices, Cutting plane methods, Branch and bound methods.

Numerical methods in optimization

*Syllabus not available*

Convex analysis and optimization

Convex analysis provides very efficient tools in every problems requiring optimization or a concept of equilibrium. We study in this course the general problem of optimizing a convex functional with or without constraint and introduce all the necessary objects (convex sets and functions, subdifferentiability, Legendre transform). We choose to present this theory in the setting of normed vector spaces

in order to reach a good level of generality, in particular infinite dimension, without requiring the use of general topological vector spaces. The course goes along the following line:

- Introduction: normed vector spaces, linear forms, separation theorems.
- Geometry of convex sets: rigidity, extremal points, the Krein-Milman theorem and its consequences in linear programming.
- Convex functions: definitions and regularity, subdifferential, Fenchel transform.
- Convex optimization and duality theory : existence theory, optimality condictions, the Fenchel-Rockafellar theorem.

Decision making: foundations

This course is an introduction to the different aspects of decision making, and need no prerequisite on this topic. Its aim is to give a solid background on the domain, to put in perspective the different areas of decision making, and especially to make the student aware of the hidden difficulties in any naïve approach to decision making. The course however remains general, and does not go deeply into each subdomain of decision making like decision under uncertainty and multicriteria decision making, for which more specialized courses exist. The course is divided as follows:

- Preference relations, preorders, semi-orders and interval orders; measurement theory, notion of scale.
- Decision under uncertainty and risk: expected utility theory, Savage model, Ellsberg and Allais paradoxes, rank dependent utility.
- Social choice theory, multiperson decision making: electing systems, Arrow's theorem
- Multiobjective decision making and multicriteria decision making: multiattribute utility theory, ELECTRE methods, multiobjective optimization.

Decision under uncertainty

This course is an introduction to decision theories under uncertainty, where the outcomes of the various possible decisions are unknown to the Decision-Maker (DM). In some cases, the relevant probabilities are objectively given to the DM. In some other cases, the DM is able to formulate subjective probability judgements. In less favorable cases, the DM is unable to do so, or refuses to make decisions based on subjective probabilities.

This course studies the three types of situations. It focuses on theoretical and axiomatic considerations as well as on economic and financial applications.

Program:

- The expected utility criterion under risk.
- Risk aversion and comparative risk aversion. Application to portfolio selection and insurance demand.
- Stochastic dominance. Prudence. Application to a consumption vs. savings problem.
- The von Neumann and Morgenstern theorem.
- Subjective probability: the de Finetti theorem and the Savage theorem.
- Ambiguity and the Ellsberg paradox.
- Capacities and the Choquet Expected utility criterion. Axiomatic characterization.
- Convex capacities and ambiguity aversion. Application to the no-trade interval.

## Information, design et markets

*Syllabus not available*

## General equilibrium theory

General equilibrium theory studies the interactions among heterogeneous agents on commodity markets. The course studies the Arrow-Debreu Model of an economy. It will consider the optimality of equilibrium, the existence of equilibrium, the local uniqueness and the structure of the set of equilibrium. The course is a necessary step to face the advanced questions coming from financial markets, markets imperfections like externalities, imperfect competition or increasing returns to scale.

## Game theory

This course presents the following concepts:

- Strategic form and dominance concepts
- Nash Equilibrium
- Zero-Sum games, value, optimal strategies
- Perfect information extensive form
- Mixed strategies
- General Information extensive form
- Sequential rationality
- Repeated games of complete information

## Statistical learning

*Syllabus not available*

## Computer training: C++

*Syllabus not available*